

Note on "Sonic Mach cones induced by fast partons in a perturbative quark-gluon plasma"

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Abstract

We make remarks on Neufeld *et al.*'s [*Phys. Rev. C* 78, 041901(R) (2008)] paper especially about the Mach cone formation. We argue that the original bow shock structure (as a fast parton moving through a quark-gluon plasma) has been smeared out after the approximations made by Neufeld *et al.*

PACS numbers: 12.38.Mh, 25.75.Ld, 25.75.Bh

Neufeld *et al.* recently presented a solution obtained from the linearized hydrodynamical equations of the medium (a fast parton traversing a weakly coupled quark-gluon plasma) in which it contains a sonic Mach cone and a dissipative wake if the parton moves at a supersonic speed [1]. In fact, Casalderrey-Solana *et al.* [2] showed that if one couples an appropriately chosen supersonic sound source to a linearized hydrodynamical evolution, one can obtain a propagating Mach cone (Casalderrey-Solana has stressed that the conical flow scenario for the observed shape is a consequence of the emission of sound by a supersonic high momentum particle propagating in the quark gluon plasma [3]). The present author, however, based on some previous works [4-5], likes to argue here that the Mach cone interpretation in [1] is not complete.

In fact, the present author argues that there are shock waves occurring under the conditions discussed in [1] as the medium through which the parton propagates is not dilute but dense [2,6]. As noted in [2], the moving 'undressed' hard parton being constantly emitting gluons, which emit new ones etc., and thus the whole shower (or the core) is a complicated nonlinear phenomenon (the multiplicity of this shower grows nonlinearly with time) and the combination should obviously be treated as a macroscopic body [7-8] passing through the medium. This could be traced in [2]: *Since the velocity of the shock depends on its intensity, the cone should in fact be somewhat rounded near its top. This effect is ignored ...* (cf. the note or Ref. 9 in [2]) or *The region near the head of the jet, which we will refer to as a 'non-hydrodynamical core', ... As found in [4](R. Baier, Y.L. Dokshitzer, S. Peigne and D. Schiff, Phys. Lett. B **345**, 277 (1995); R. Baier, Y.L. Dokshitzer, A.H. Mueller and D. Schiff, JHEP **20010109** 033 (2001)), the multiplicity of this shower grows nonlinearly with time, so eventually the core may become a macroscopic body, providing a large perturbation of the matter. From the hydrodynamical point of view, its size is limited from below by the dissipative 'sound attenuation length' $\Gamma_s = (4/3)\eta/(\epsilon + p)$, with η being the shear viscosity* (page 24 of [2]; please see the details for the relevant symbols therein).

Meanwhile, there is inconsistency in the theoretical treatment in [1] which will be described below. Neufeld *et al.* solved the hydrodynamical equation (cf. Eq. (8) and the detailed explanations for each symbol in [1])

$$\partial_\mu T^{\mu\nu} = J^\nu \equiv \int \frac{d\mathbf{p} p^\nu}{(2\pi)^3} (\nabla_{p_i} D_{ij}(\mathbf{p}, t) \nabla_{p_j} f(\mathbf{x}, \mathbf{p}, t)) \quad (1)$$

by assuming that the energy and momentum density deposited by the **parton** is small

compared to the equilibrium energy density of the medium. Here, $f(\mathbf{x}, \mathbf{p}, t)$ is the ensemble averaged phase-space distribution of medium partons [1] and

$$D_{ij}(\mathbf{p}, t) = \int_{-\infty}^t dt' F_i(\mathbf{x}, t) F_j(\mathbf{x}', t'), \quad (2)$$

with $F_i(\mathbf{x}, t) = gQ^a(t)[E_i^a(\mathbf{x}, t) + (\mathbf{v} \times \mathbf{B})_i^a(\mathbf{x}, t)]$ being the color Lorentz force on a medium particle and J^ν represents a source term due to the interaction of the medium with the passing fast parton. Subsequently, to evaluate the source term in the right-hand-side of Eq. (1), Neufeld *et al.* considered a thermal plasma of massless gluons with the unperturbed distribution (cf. Eq. (12) in [1]). The present author doubts : Where is the contribution from the **entire parton** (say, quarks) [2,9-11]?

To briefly check the mathematical derivations of [1] or [10] (they were self-cited, cf. Ref. 24 in [10], i.e., [1] here or Ref. 19 in [1], i.e., [10] here), we start from equations (13) : Integral form of $J^0(\mathbf{x}, t)$ and (14) : Integral form of $J^k(\mathbf{x}, t)$. We have no idea how the term : $i\epsilon$ can be inserted into the denominator of the integrand of above mentioned integrals (cf. $(\omega' - \mathbf{k}' \cdot \hat{\mathbf{v}} + i\epsilon)$ in Eqs. (13) and (14) of [1])? As the first author for [1] and [10] is the same, then the present author tried to trace this back from [10]. Similarly, we also have no idea how the term : $i\epsilon$ can be inserted into the denominator of the integrand of the integral form for (i.e., Eq. (12) in [10])

$$f_1^a = -\frac{igC_2}{N_c^2 - 1} \int \frac{d^4k}{(2\pi)^4} \int d^4x' U_{ab}(x, x') \times \frac{e^{ik(x'-x)}}{v k + i\epsilon} \mathbf{F}^b(x') \cdot \nabla_p f_0, \quad (3)$$

$$f_1^a \equiv f^a(\mathbf{x}, \mathbf{p}, t) = \int dQ Q^a f(\mathbf{x}, \mathbf{p}, t).$$

Please refer the detailed explanation for symbols or notations appeared in above expression to [10]. However, there are no definition for $i\epsilon$ up to the relevant statements near Eq. (12) of [10]? Even though *A detailed derivation of the source term, including color screening by the medium, is presented in Ref. [19]* appeared in [1]?

Meanwhile we know that Mach cones are V-shaped disturbances produced by a supersonic object [7-8] or the interference of sound waves from a supersonic source leads to the Mach cone [3]. They are familiar in gas dynamics [7-8]. The cone's Mach angle is $\theta = \sin^{-1} 1/M$, where $M = u/c$ is the Mach number of an object moving at speed u through a medium with an acoustic speed c . As we argued above that there are shock waves appearing in [1]. The disappearing of shock waves in [1] could be traced in the following : *Since hydrodynamics is*

only valid at distances that are large compared to the mean free path, and in a weakly coupled plasma the mean free path is parametrically large compared to the color screening length, the source term generated by an energetic parton is, in first approximation, point-like. In this spirit, the source term [Eqs. (15)-(18)] derived here can be thought of as a sophisticated representation of a δ function \dots . It means the strong nonlinearity due to the propagating blunt-body-like [7] parton has been smeared out due to point-like treatments in [1]. Similar smearing-out could be traced by ... We evaluate each term by boosting to a frame comoving with our volume element and then exploiting the assumption of local thermal equilibrium. Note that near the shock as there is a discontinuity and entropy condition [5,7], it is not at (thermal) equilibrium.

The originally curved **bow shocks** [7-8,12] have been replaced and approximated by linear V-shaped (Mach) wave patterns [2,7-8]. Furthermore, to remind the readers, as mentioned in [1] : *We will incorporate the effect of color screening and short-distance quantum effects by appropriate infrared and ultraviolet cutoffs.*, considering above both statements, how can the authors of [1] capture the detailed long-wave (larger length-scale) limit or hydrodynamical behavior (say, shock structures) by only a short-distance (smaller length-scale) treatment and simultaneously neglecting the highly nonlinearities (we well as the singularities due to the core or 'macroscopic body' [2]) by using a set of cutoffs?

Finally, to be precise, considering the analogy from the dusty plasma (cf. Samsonov *et al.* in [8]), due to the finite size of the Debye sphere surrounding the fast-moving parton, the vertex of the Mach cone is rounded rather than pointed. From figures 1, 2 and 3 of [1], we cannot observe this Debye sphere? The important information about the opening angle of the Mach cone [8] is also absent in [1].

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